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ABSTRACT

We report the experimental measurement and simulation of the transient electrostatic pull-in characteristics of micro-structural beams fabricated with silicon surface micromachining. Pull-in dynamics are investigated under the influence of compressible squeezed-film damping (CSQFD) for large amplitude motion. A linearized one-dimensional model, using a fitted damping constant, and a two-dimensional finite-difference model, based on the compressible isothermal Reynold's equation, are used to simulate the experimental data.

INTRODUCTION

Elastically supported microstructures become unstable under a nonlinear electrostatic force beyond an applied voltage called the pull-in voltage. In mechanically static situations, this voltage is termed the static pull-in voltage (V_{PI}). Models for V_{PI} have been used for extraction of material properties and for process monitoring from microelectromechanical system (MEMS) test structures. [1-4]

In mechanically dynamic situations, MEMS devices over small air gaps are subject to CSQFD. This effect has been simulated for large amplitude motion by Yang and Senturia using the nonlinearized form of the compressible isothermal Reynold's equation. [5] Here we investigate the coupled elastomechanical-electrostatic-CSQFD problem for the large dynamic motion of microstructural fixed-fixed beams by measuring their pull-in times after step voltage applications.

EXPERIMENTS AND SIMULATIONS

Two fixed-fixed polysilicon beams, 610 µm long and 710 µm long, 2.12 µm thick and 40 µm wide over a 2.07 µm gap are placed in a circuit similar to that in Figure 1. [6] A zero-to-peak step input voltage is applied between the beam and a fixed bottom conductor. Similar to that shown in Figure 2, the pull-in time is measured as the delay between the application of the step input and the sharp increase in V_{out} after the beam makes contact to the conductor.

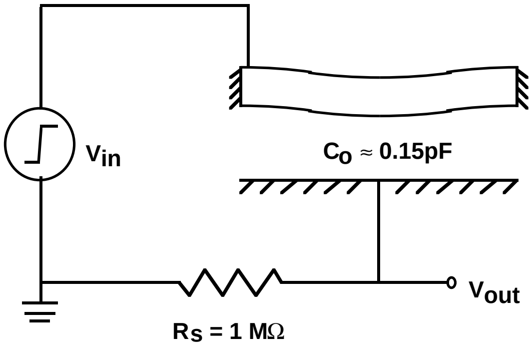


Fig. 1: Circuit used for measuring pull-in times.

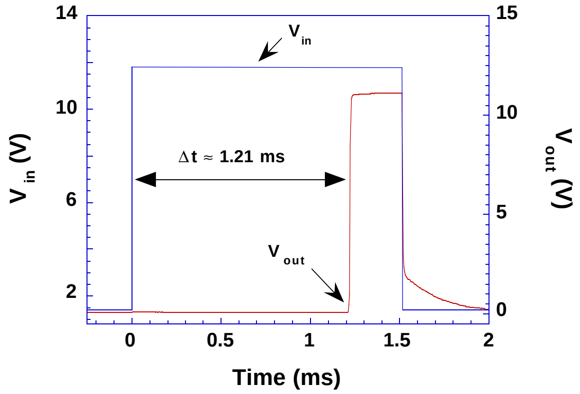


Fig. 2: Oscilloscope data showing Δt, which measures the pull-in time from a static deflection near 1.3 V to pull-in after application of 11.8 V.

Two approaches were taken to model the pull-in time. The first uses a lumped mechanical circuit-equivalent (1D macro-model), similar to the distributed models in [7] for analyzing small amplitude motion of accelerometers. Our model is shown in Figure 3. The beam is a lumped mass *m* and moves along the *x*-coordinate, *k* is a linearized spring constant and *b* is a linearized damping factor. The electrostatic force is derived from the voltage across the parallel plate capacitance with a gap $h = (g-x)$, where *g* is the undeflected gap.

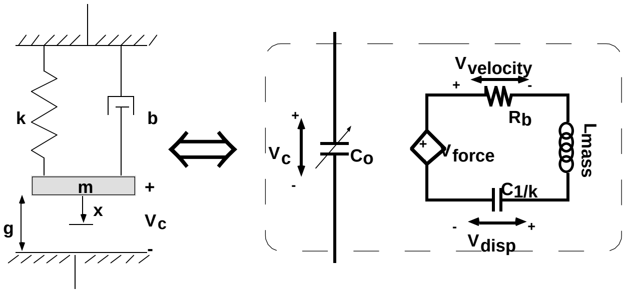


Fig. 3: A mechanical circuit-equivalent of the 1D pull-in macro-model. R_b, L_{mass}, and $C_{1/k}$ have components equal to their MKS subscript values in the base units of Ω, H, and F, respectively. $C_o = \varepsilon_o A / (g-V_{disp})$ and $V_{force} = 1/2\ \varepsilon_o A V_c^2 / (g-V_{disp})^2 \bullet (1/k)$.

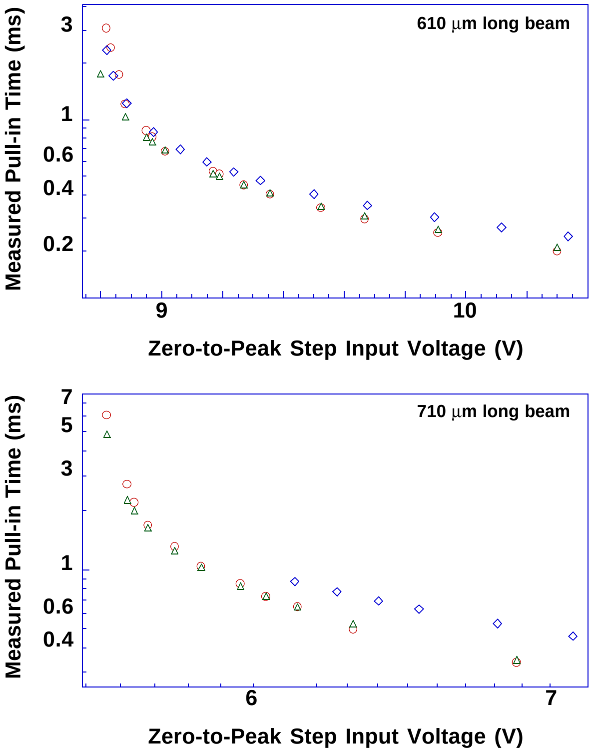
The second approach solves the two-dimensional (2D) finite-difference equation of motion for a uniformly-flat Euler wide-beam (1) coupled to a modified Reynold's equation (2) through a damping pressure P_D [5,8] and to a parallel-plate approximation of the nonlinear electrostatic pressure P_E:

$$\frac{\tilde{E}t'^3}{12}\nabla^4 h + \sigma_s(h)\nabla^2 h = P_E + P_D - \rho_s \frac{\partial^2 h}{\partial t^2}. \quad (1)$$

$$\frac{\partial(hP_D)}{\partial t} = \frac{\nabla\cdot\left(P_D h^3 \nabla P_D\right)}{12\,\mu}. \qquad (2)$$

The air viscosity μ is 1.82×10^{-5} kg/(m•s). t' is the beam thickness. $\tilde{E}$ = 164 GPa is the plate modulus and $\sigma_s(h)/t'$ is the gap-dependent stress, where $\sigma_s(h{=}g)/t'$ = -3.5 MPa as determined from the models for V_{PI}. [1] The density of polysilicon, ρ_s/t' is 2200 kg/m^3.

Figures 4 & 5 show the measured and simulated pull-in times for both beams in air. Independent static measurements of V_{PI} for the 610 µm and 710 µm beams indicate they are 8.76 V and 5.54 V, respectively. Both are close to the lowest recorded values for which pull-in times could be measured.



Figs. 4 & 5: Measured pull-in times in air (circles). 1D (triangles) and the 2D (diamonds) simulations are shown for both the 610 µm and 710 µm beams.

For the 1D model, k is chosen by matching the 1D equation for $V_{PI} = (8kg^3/27\varepsilon_o A)^{1/2}$ to the statically measured V_{PI}. With gaps of g = 2.07 µm and conductor areas of A_{610} = 2.44×10^4 µm^2 and A_{710} = 2.84×10^4 µm^2, the 610 µm and 710µm beam have k's equal to 6.33 N/m and 2.94 N/m, respectively. m is determined from the beam geometry and from ρ_s, giving masses of 12 ng and 14 ng, for the two beams. b is determined by finding a best fit to the experimental data using the 1D model. This yields $b = 35 \pm 2$ µg/s for both beams.

In Figure 6, we plot the measured and simulated pull-in times for the 610 µm beam in vacuum at 6×10^{-3} mbar. (Note, 1 atm ≈ 1000 mbar.) The times are an order of magnitude shorter than the air-damped case in Fig 4. Because of inertial effects, the beam can pull in dynamically at voltages below V_{PI}, an effect predicted by the 1D model with $b = 0$. When calculated using the k reported above, this dynamic pull-in occurs at 8.04 V, which agrees with the measured pull-in voltage of 8.10 V and the corresponding simulated pull-in voltage of 8.04 V from the 2D model.

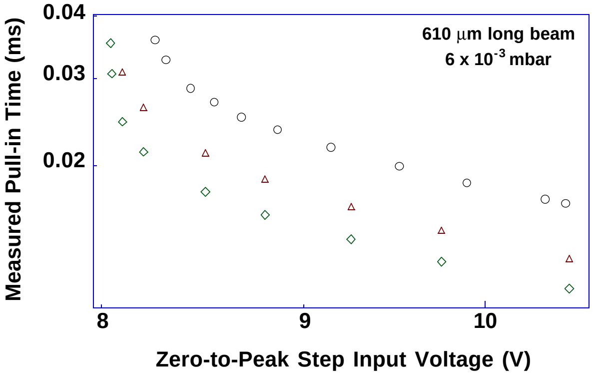


Fig. 6: Measured pull-in times at 6×10^{-3} mbar for the 610 µm beam. Vacuum simulations in 1D (b=0, triangles) including a 40 pF parasitic capacitance on V_{out}, and in 2D (P_D = 0, diamonds) are also shown.

CONCLUSIONS

We find that the 1D and 2D simulations generally model the pull-in time data from our beams. Additional effects to be included are (1) the breakdown of the continuum flow approximation [9], (2) fringing fields, (3) more precise material constants ($\tilde{E}$ and $\sigma_s(h{=}g)$), and (4) structural damping.

ACKNOWLEDGMENTS

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