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ELECTROSTATICALLY-ACTUATED BEAMS

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ABSTRACT

We report the experimental measurement and simulation of the transient electrostatic pull-in characteristics of microstructural beams fabricated with silicon surface micromachining. Pull-in dynamics are investigated under the influence of compressible squeezed-film damping (CSQFD) for large amplitude motion. A linearized one-dimensional model, using a fitted damping constant, and a two-dimensional finite-difference model, based on the compressible isothermal Reynolds' equation, are used to simulate the experimental data.

INTRODUCTION

Elastically supported microstructures become unstable under a nonlinear electrostatic force beyond an applied voltage called the pull-in voltage. In mechanically static situations, this voltage is termed the static pull-in voltage \( V_{PI} \). Models for \( V_{PI} \) have been used for extraction of material properties and for process monitoring from microelectromechanical system (MEMS) test structures. [1-4]

In mechanically dynamic situations, MEMS devices over small air gaps are subject to CSQFD. This effect has been simulated for large amplitude motion by Yang and Senturia using the nonlinearized form of the compressible isothermal Reynolds' equation. [5] Here we investigate the coupled elastomechanical-electrostatic-CSQFD problem for the large dynamic motion of microstructural fixed-fixed beams by measuring their pull-in times after step voltage applications.

EXPERIMENTS AND SIMULATIONS

Two fixed-fixed polysilicon beams, 610 \( \mu \)m long and 710 \( \mu \)m long, 2.12 \( \mu \)m thick and 40 \( \mu \)m wide over a 2.07 \( \mu \)m gap are placed in a circuit similar to that in Figure 1. [6] A zero-to-peak step input voltage is applied between the beam and a fixed bottom conductor. Similar to that shown in Figure 2, the pull-in time is measured as the delay between the application of the step input and the sharp increase in \( V_{out} \) after the beam makes contact to the conductor.

\[ V_{in} \]

\[ Rs = 1 \text{ M}\Omega \]

\[ 0 V_{out} \]

\[ C_0 = 0.15pF \]

Fig. 1: Circuit used for measuring pull-in times.

Two approaches were taken to model the pull-in time. The first uses a lumped mechanical circuit-equivalent (1D macro-model), similar to the distributed models in [7] for analyzing small amplitude motion of accelerometers. Our model is shown in Figure 3. The beam is a lumped mass \( m \) and moves along the \( x \)-coordinate, \( k \) is a linearized spring constant and \( b \) is a linearized damping factor. The electrostatic force is derived from the voltage across the parallel plate capacitance with a gap \( h = (g-x) \), where \( g \) is the undeflected gap.

\[ \Delta t = 1.21 \text{ ms} \]

\[ V_{out} \]

\[ V_{in} \]

\[ \Delta t \]

\[ V_{displ} \]

\[ V_{velocity} \]

\[ V_{force} \]

\[ V_{out} \]

Fig. 2: Oscilloscope data showing \( \Delta t \), which measures the pull-in time from a static deflection near 1.3 \( V \) to pull-in after application of 11.8 \( V \).

The second approach solves the two-dimensional (2D) finite-difference equation of motion for a uniformly-flat Euler wide-beam (1) coupled to a modified Reynolds’ equation (2) through a damping pressure \( P_D \) [5,8] and to a parallel-plate approximation of the nonlinear electrostatic pressure \( P_E \):

\[ \frac{\tilde{E}t^{1/2}}{12} \nabla^4 h + \sigma_s (h) \nabla^2 h = P_E + P_D - \rho_s \frac{\partial^2 h}{\partial t^2}. \]

(1)
\[ \frac{\partial (h P_D)}{\partial t} = \nabla \cdot \left( P_D h^2 \nabla P_D \right). \]  

(2)

The air viscosity \( \mu \) is \( 1.82 \times 10^{-5} \) kg/(m·s). \( t' \) is the beam thickness. \( \dot{E} = 164 \) GPa is the plate modulus and \( \sigma(h)/t' \) is the gap-dependent stress, where \( \sigma(h=g)/t' = -3.5 \) MPa as determined from the models for \( V_{P1} \). [1] The density of polysilicon, \( \rho_s \), is \( 2200 \) kg/m\(^3\).

Figures 4 & 5 show the measured and simulated pull-in times for both beams in air. Independent static measurements of \( V_{P1} \) for the 610 \( \mu \)m and 710 \( \mu \)m beams indicate they are 8.76 V and 5.54 V, respectively. Both are close to the lowest recorded values for which pull-in times could be measured.

\[ \text{Fig. 6: Measured pull-in times at } 6 \times 10^{-3} \text{ mbar for the } \text{610} \mu \text{m beam. Vacuum simulations in } 1D \text{ (} b=0, \text{ triangles) including a } 40 \text{ pF parasitic capacitance on } V_{\text{out}}, \text{ and in } 2D \text{ (} P_D = 0, \text{ diamonds) are also shown.} \]

CONCLUSIONS

We find that the 1D and 2D simulations generally model the pull-in time data from our beams. Additional effects to be included are (1) the breakdown of the continuum flow approximation [9], (2) fringing fields, (3) more precise material constants (\( \dot{E} \) and \( \sigma(h=g) \)), and (4) structural damping.

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